NCC-0030-492002 Seat No. ________

M. Sc. (Applied Physics) (Sem. II) (CBCS) Examination

April / May – 2017

Applied Mathematics : Paper-6

Faculty Code : 0030
Subject Code : 492002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) **All** the questions are **compulsory**.
(ii) Give **calculations** if required.

1 Answer any **seven** : **14**

(1) Write general form of linear differential equation of higher order with constant coefficients.

(2) Write the corresponding and auxiliary equation of the differential equation, $\left(D^2-1\right)y = x\sin(3x)+\cos(x)$.

(3) Define : Complementary function (C.F.)

(4) Define : Particular integral (P.I.)

(5) Find $\frac{\partial z}{\partial x}$ & $\frac{\partial z}{\partial y}$ for $z = x^2y^3 + xy$.

(6) State conditions for a bivariate function having maxima.

(7) State Lagrange's linear equation.

(8) State one dimensional wave equation.

(9) State Rolle's Mean Value Theorem.

(10) Define : Jacobian.

2 (a) Answer the following : **4**

(1) Solve : $\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=0$.

(2) Explain the meaning of reciprocal of the differential operator D.

(b) Prove that $\frac{1}{f(D)}e^{ax} = \frac{1}{f(a)}e^{ax}$, where $f(a) \neq 0$. **5**

(c) Solve : $\left(D^2 - 2D\right)y = e^x \sin x$. **5**

OR

2 (a) Answer the following : **4**

(1) Solve : $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$.

(2) Show that $\frac{1}{D-a}X = e^{ax}\int Xe^{-ax}dx$.

(b) Prove that $\frac{1}{f\left(D^2\right)}\sin(ax+b) = \frac{1}{f\left(-a^2\right)}\sin(ax+b)$, **5**

if $f\left(-a^2\right) \neq 0$.

(c) Solve : $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = \cos^2 x$. **5**

3 (a) Answer the following : **4**

(1) Find $\frac{\partial z}{\partial x}$ & $\frac{\partial z}{\partial y}$ if $z = \log\left(x^2 + y^2\right)$.

(2) Show that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ if $z = x^3 + y^3 - 3xy$.

(b) State and prove the Euler's theorem for a homogeneous function for two variables. **5**

(c) If $f = (y-z, z-x, x-y)$, then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **5**

OR

3 (a) Answer the following : **4**

(1) Find $\frac{\partial z}{\partial x}$ & $\frac{\partial z}{\partial y}$ if $xy + yz + zx = 1$.

(2) If $W = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, then prove that

$$x\frac{\partial W}{\partial x} + y\frac{\partial W}{\partial y} + z\frac{\partial W}{\partial z} = 0.$$

(b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, then prove that **5**

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin(4u) - \sin(2u).$$

(c) If $J = \frac{\partial(u,v)}{\partial(x,y)}$ & $J' = \frac{\partial(x,y)}{\partial(u,v)}$ then prove that, $JJ' = 1$. **5**

4 (a) Answer the following : **4**

(1) Derive the PDE from $z = xy + f\left(x^2 + y^2\right)$.

(2) Derive the PDE of all spheres of radius 3 units having their centres on the XY-plane.

(b) Solve : $\frac{\partial^3 z}{\partial x^2 \partial y} + 18xy^2 + \sin(2x - y) = 0$. **5**

(c) Solve : $pyz + qzx = xy$. **5**

OR

4 (a) Answer the following : **4**

(1) Derive the PDE from $f(x - z, y - z) = 0$.

(2) Derive the differential equation of all spheres whose centres lie on the Z-axis.

(b) Solve : $\dfrac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x+3y)$. **5**

(c) Solve : $(mz-ny)\dfrac{\partial z}{\partial x} + (nx-lz)\dfrac{\partial z}{\partial y} = (ly-mx)$. **5**

5 (a) Answer the following : **4**

(1) State Lagrange's mean value theorem.

(2) Verify Rolle's mean value theorem for $f(x) = x^3 - 4x$ on $[-2, 2]$ and find the value of c if possible.

(b) Expand $f(x) = \sin(x)$ in the ascending powers of x. **5**

(c) Find the area of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1,\ (b < a)$ by using double integration. **5**

OR

5 (a) Answer the following : **4**

(1) State Cauchy's mean value theorem.

(2) Verify Lagrange's mean value theorem for $(x) = \log(x)$ in $\left[\dfrac{1}{2}, 2\right]$.

(b) Expand $f(x) = \log(x)$ in the ascending powers of. **5**

(c) Evaluate : $\int_0^1 \int_0^1 \int_0^1 (x+y+z)\,dxdydz$. **5**